REDUCTION OF A 2*n*x2*n* HAMILTONIAN-MATRIX EIGENPROBLEM TO A *n*x*n* STANDARD EIGENPROBLEM

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A novel approach for solving the real-Hamiltonian-matrix eigenproblem, which occurs in variational theories of mechanics and control theory, is introduced in this paper. This new method reduces a 2*n*x2*n* real-Hamiltonian-matrix eigenproblem to a *n*x*n* standard, real-matrix eigenproblem, which can be solved using existing QR Householder algorithms. The computational time of a QR method is generally considered to be proportional to the cube of the matrix dimension, thus this new method should be considerably more time efficient. The eigenvalues and eigenvectors of the Hamiltonian-matrix eigenproblem are computed using the eigenvalues and eigenvectors of the standard eigenproblem. The solution technique uses symplectic, orthogonal; symplectic non-orthogonal; and non-symplectic; transformation matrices to transform the Hamiltonian matrix such that the associated algebraic Ricatti equation is satisfied by the identity matrix. As an example, the new method is incorporated into the computer program that implements Pagano's axisymmetric cylinder model.

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